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Torus Modelling

(and other approximations)

With: James Binney, Jason Sanders

Orbits are the building blocks of galaxies

Describing a star as being at x, with velocity v is unhelpful – it will change

Better: describe as on orbit labelled J at point θ. J stays ~fixed.

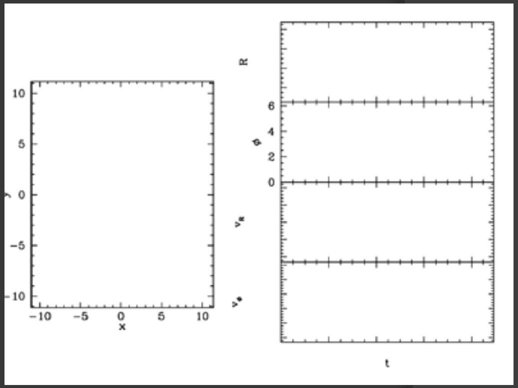
Jeans' theorem: A steady state df f(x,v) is f(J). 6D structure -> 3D. Only way to find Φ for near steady-state systems.

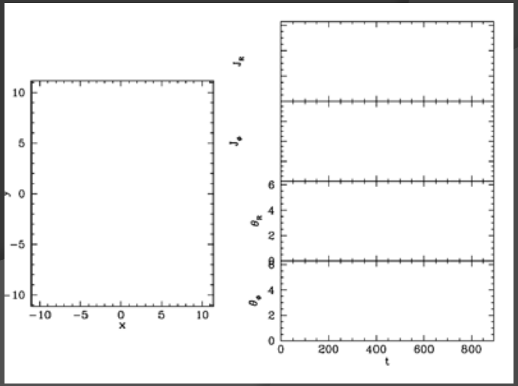
To describe a galaxy/model describe structure in J

Actions – dynamicists love them!

$$J_i = \frac{1}{2\pi} \int_{\gamma_i} \mathbf{p} \cdot \mathrm{d}\mathbf{q}$$

- Adiabatically invariant
- They can be used as momenta in canonical coordinates
- Conjugate variables, $\underline{\theta}$, increase linearly with time, so dynamics is easy.
- Reasonably intuitive (J_R, J_z, J_ϕ range 0 to ∞)
- Natural coordinates of perturbation theory





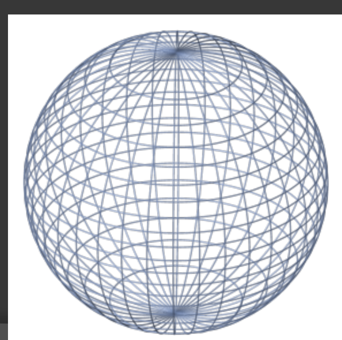
The problem

We can only find them analytically for the isochrone potential.

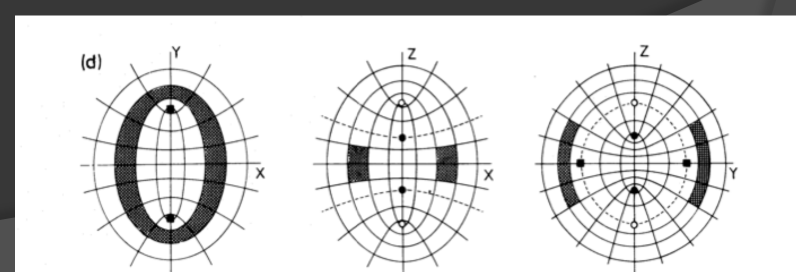
$$\Phi_{\rm iso} = \frac{-GM}{b + \sqrt{b^2 + r^2}}$$

Using 1D numerical integrals:

Any spherical potential



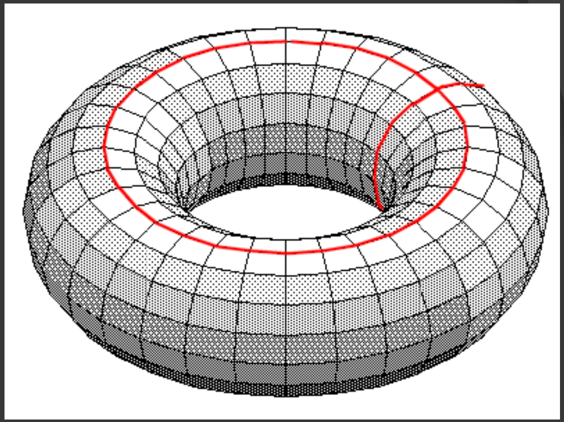
Stäckel potential (separable in ellipsoidal coordinates).



The solutions: 1. Torus modelling

Why torus?

1-torus is a circle
2-torus is the surface of a doughnut
An orbit is a 3-torus in (6D) phase-space



Torus modelling (McGill & Binney 1990) – We can distort the tori in a "toy" potential (isochrone) into our Galactic potential

Ensure that distortion retains characteristics of toy torus (through use of appropriate "generating function") and is at constant H (or, at least, minimise variation).

For a single value of $\underline{J}$, gives $\underline{x}(\underline{\theta})$, $\underline{v}(\underline{\theta})$

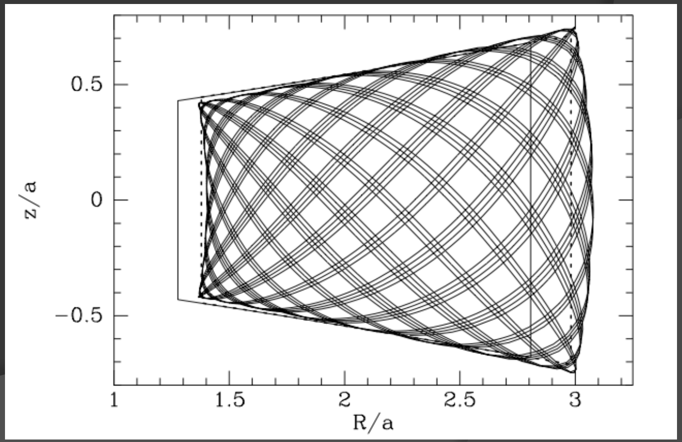
e.g. McMillan & Binney (2008)

The solutions: 2. Adiabatic approximation

- Motion near Galactic plane is ~separable in R,z
- Approximate z-motion as conserving J_z calculated as 1D integral in

$$\Psi_z(z;t) = \Phi[R(t), z]$$

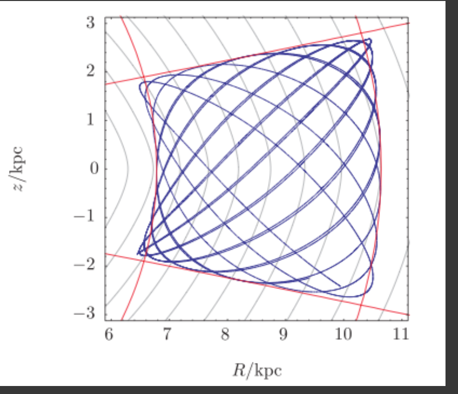
- Works OK for disc
- Gives J(x,v)
- Tilt of velocity ellipsoid = 0



e.g. Binney & McMillan (2011)

The solutions: 3. Stäckel fitting

Equations of motion in a Stäckel potential are separable in ellipsoidal coordinates. This makes it easy to calculate all 3 actions.



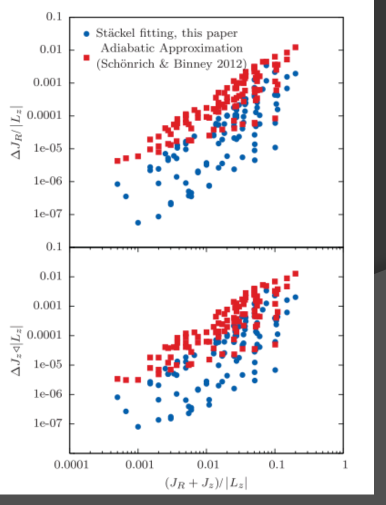
So, take orbit in true potential and fit a Stäckel potential in the volume that the orbit probes.

Calculate actions in this Stäckel potential.

Gives J(x,v) and θ(x,v)

More accurate than adiabatic approximation

Somewhat slow and unwieldy



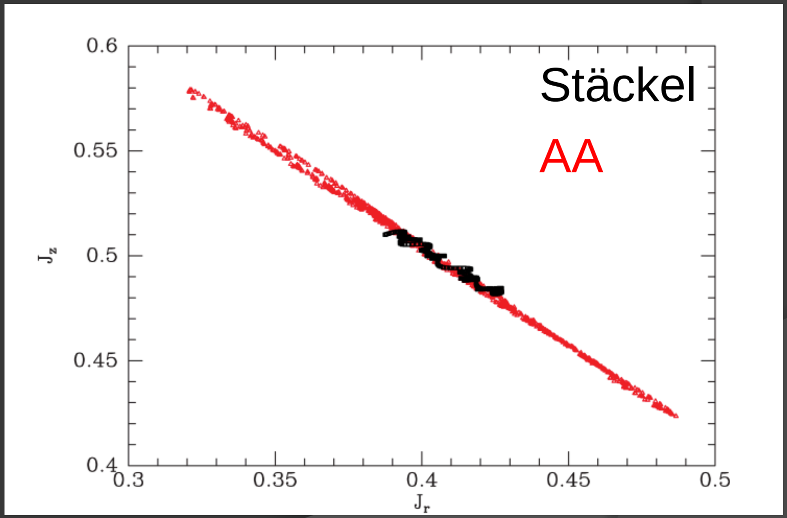
Sanders (2012)

The solutions: 4. Stäckel “fudge”

Again, relies on assumption that Φ is similar to Stäckel potential.

Pick one shape for the Stäckel potential (coordinate system u,v)

Given ($\underline{x}$,$\underline{v}$), find (u,v,p_u,p_v), do some numerical trickery, and get out actions via 1D integral (or interpolation on table of E, L_z and complicated function of u or v)



More accurate than AA
Velocity ellipsoid tilt (put in by hand)
Fast

Binney (2012)

Distribution function f(J)

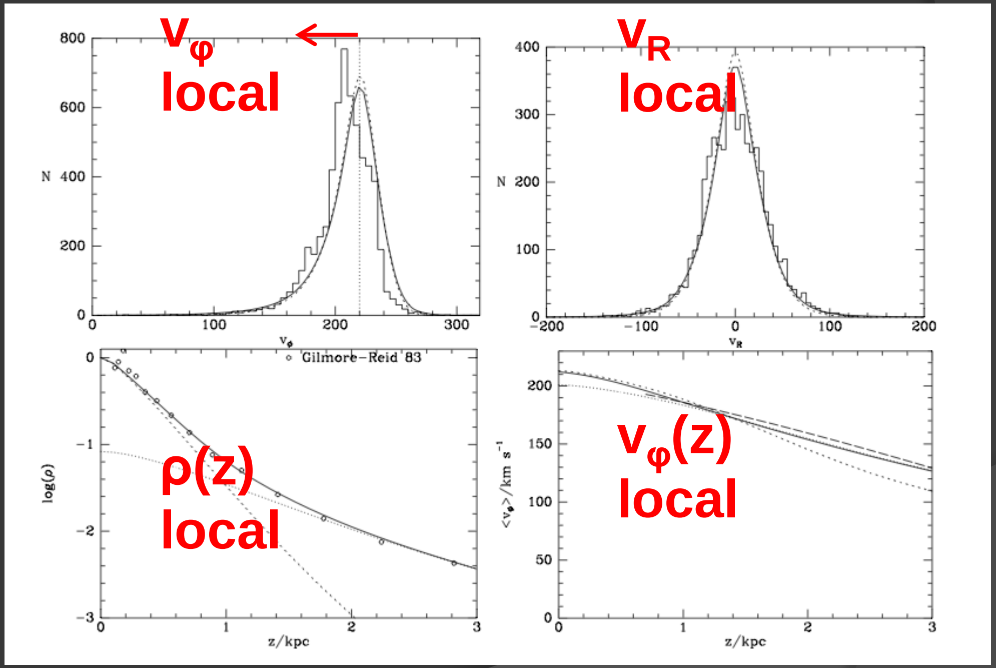
Need a df for the disc, a simple choice:
(in keeping with past ideas e.g. Shu 1969)

$$f(\mathbf{J}) \propto \Sigma(R_c(J_\phi)) \prod_{i=z,r} \exp(-\frac{\omega_i J_i}{\sigma_i^2})$$

"quasi-isothermal"

Can be used to provide good fits to local kinematics and density structure (Binney 2010, see also Bovy's MAPs)

Indeed they can point out false assumptions ($V_\odot$ wrong by ~7km/s – see also McMillan & Binney 2010, Schönrich, Binney & Dehnen 2010)



Finding the Galactic potential

Key aim of many Galactic surveys (RAVE, Gaia…)

Only way to determine dark matter distribution.

Data for Milky Way are different from those for external galaxies – more precise, more dimensions, far from physical quantities of interest (parallax, $\boldsymbol{\mu}$, v_{los},…)

Assume we can describe stars as f($\underline{J}$) in some potential, then maximise P(observations | f($\underline{J}$)) for each potential (bearing in mind selection effects)

Consider two methods:

1. Using an torus (orbit) library – should be more suitable than Schwartzchild
2. Finding $\underline{J}(\underline{x},\underline{v})$ approximately using Stäckel fudge.

What are we doing (numerically)

$$P(\mathrm{observation}|\mathrm{Model}) = \int P(\mathrm{observation}|\mathbf{x},\mathbf{v}) \times P(\mathbf{x},\mathbf{v}|\mathrm{Model})\, \mathrm{d}^3\mathbf{x}\, \mathrm{d}^3\mathbf{v}$$

Non-negligible for very small volume in phase space

f(J) in Φ

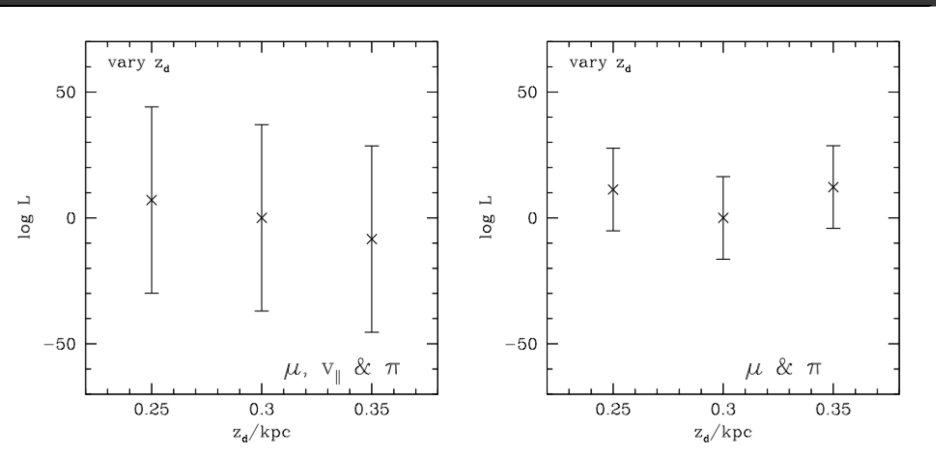
If one does this integral with an orbit library (evaluate at δ-functions in J), the number of relevant orbits for a given observation is small.

When you change Φ, number of relevant orbits changes in uncontrolled way – shot noise.

If instead you fix x,v at which you evaluate integral, this noise is greatly reduced

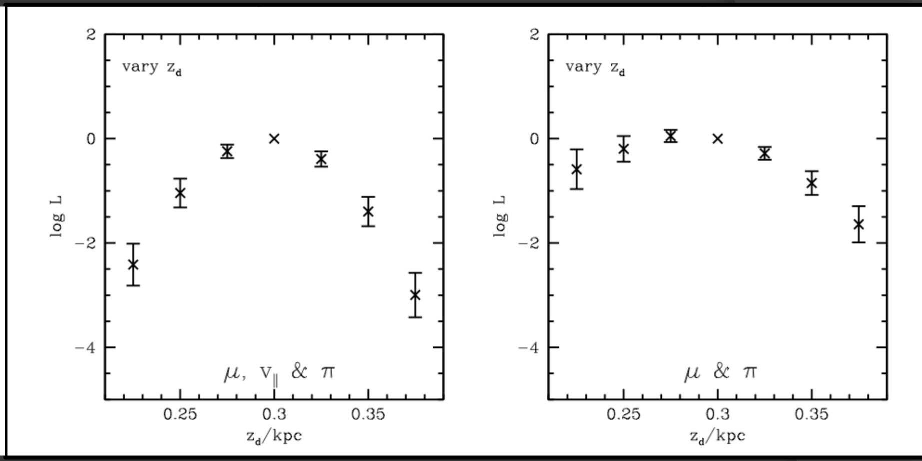
Don’t use an orbit library!

Torus library



N.B. change of scale

Calculation of J(x,v)



Error bars: numerical uncertainty

Data are too precise.

They slip through the gaps in an orbit library.

McMillan & Binney (2013)

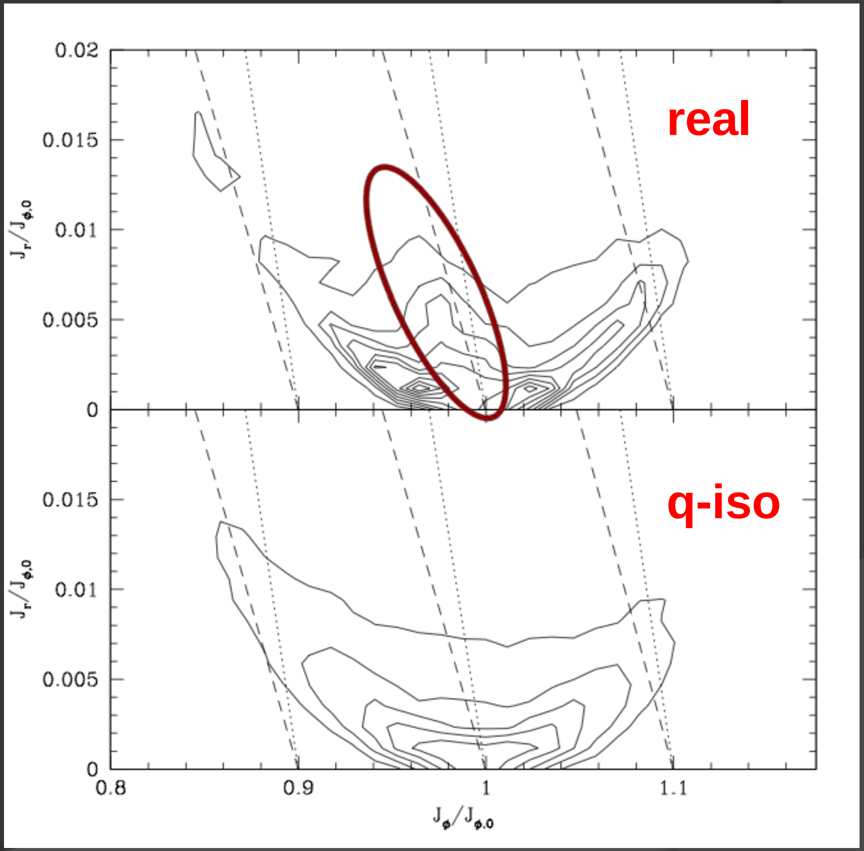
Adding effects of trapping at Lindblad resonances

Quasi-isothermal df is very smooth

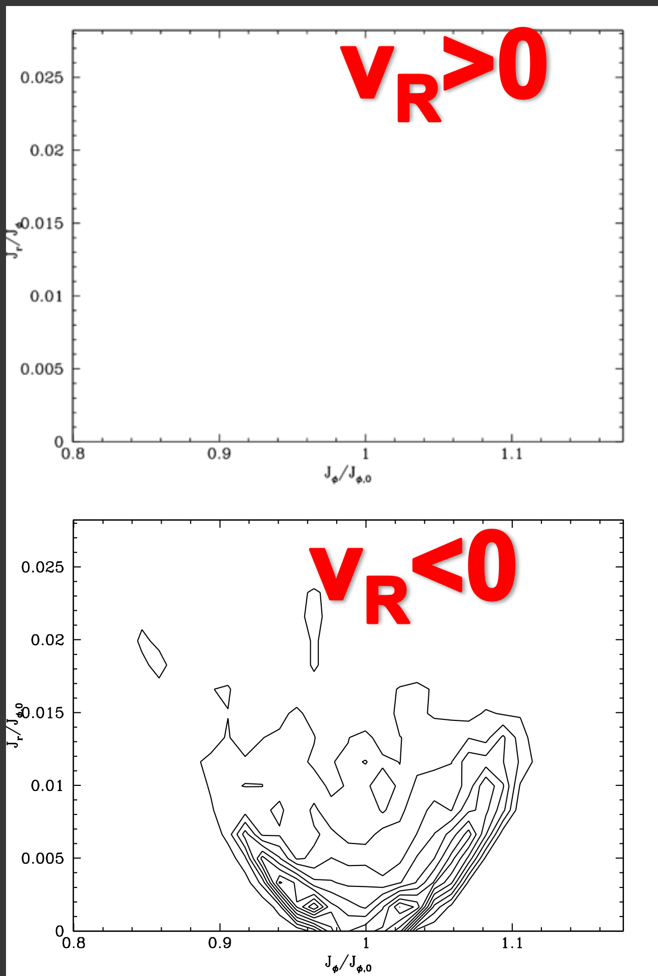
The SN velocity distribution is not.

The Hyades can be explained by a Lindblad resonance (Sellwood 2010, McMillan 2011)

$$m(\Omega_\mathrm{p} - \Omega_\phi) = l\Omega_R$$



Lindblad resonances cont.

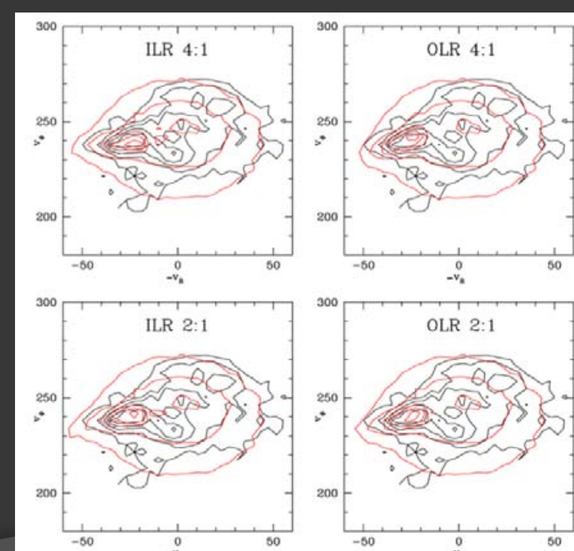


Modelled as trapping near combination of actions, and combination of angles.

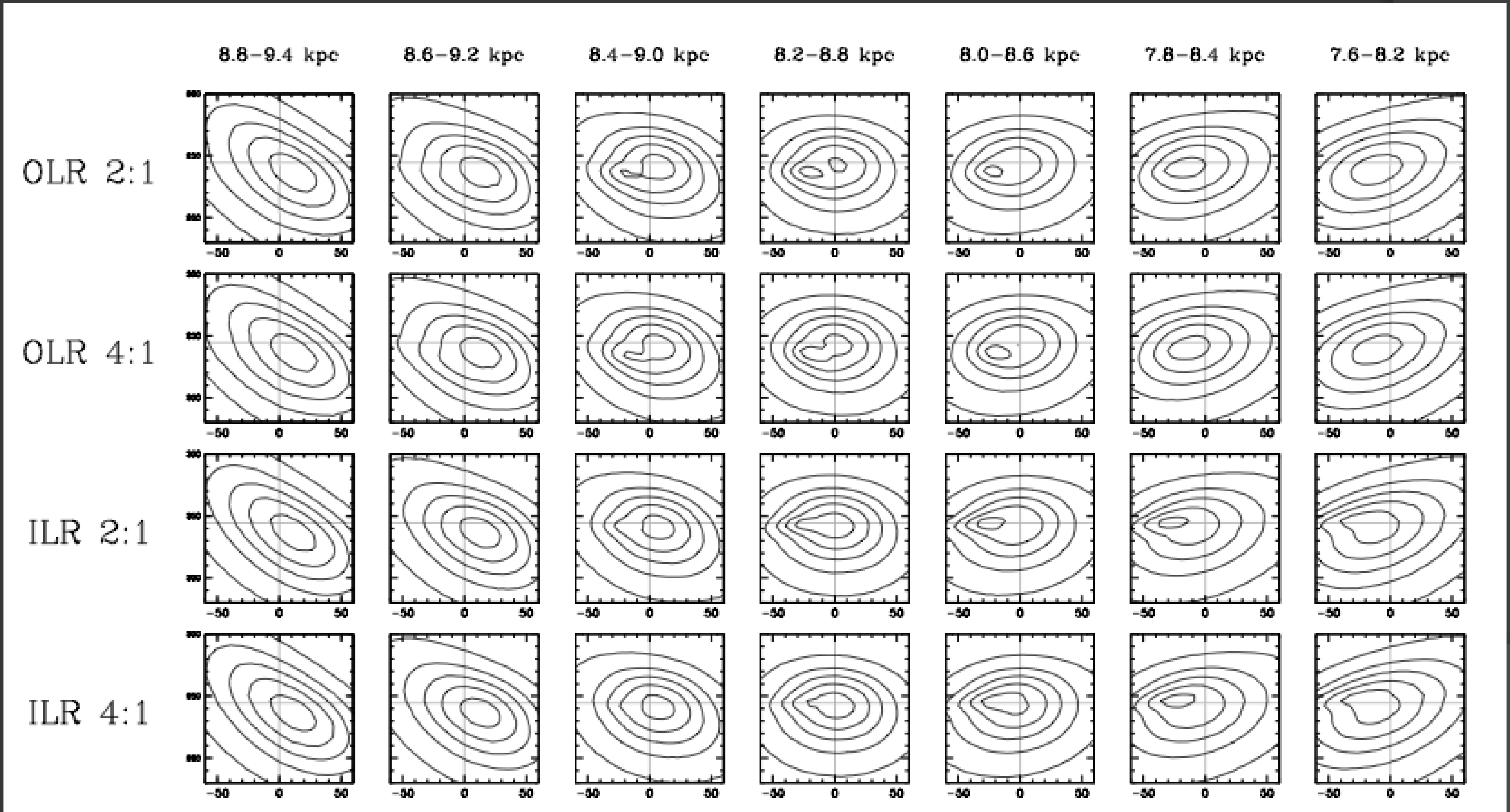
$$l\theta_r + m\theta_\phi \simeq \text{const},$$

If no angle dependence, symmetric w.r.t. v_R

Which resonance? Nasty selection effects mean that we need to look further away.



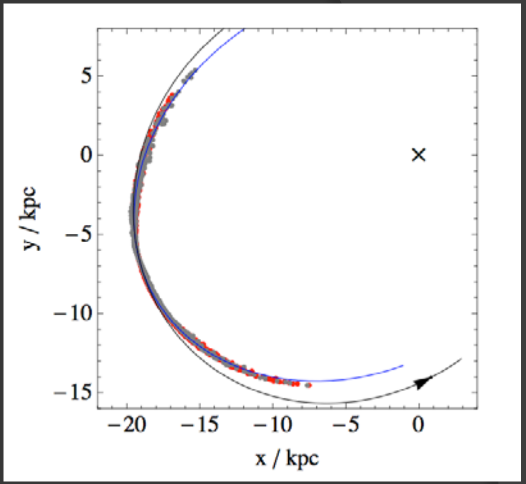
Further away (RAVE volume)



Differences clear in RAVE volume, but not once errors added (c.f. Antoja et al 2012)

McMillan (2013)

Streams (briefly)

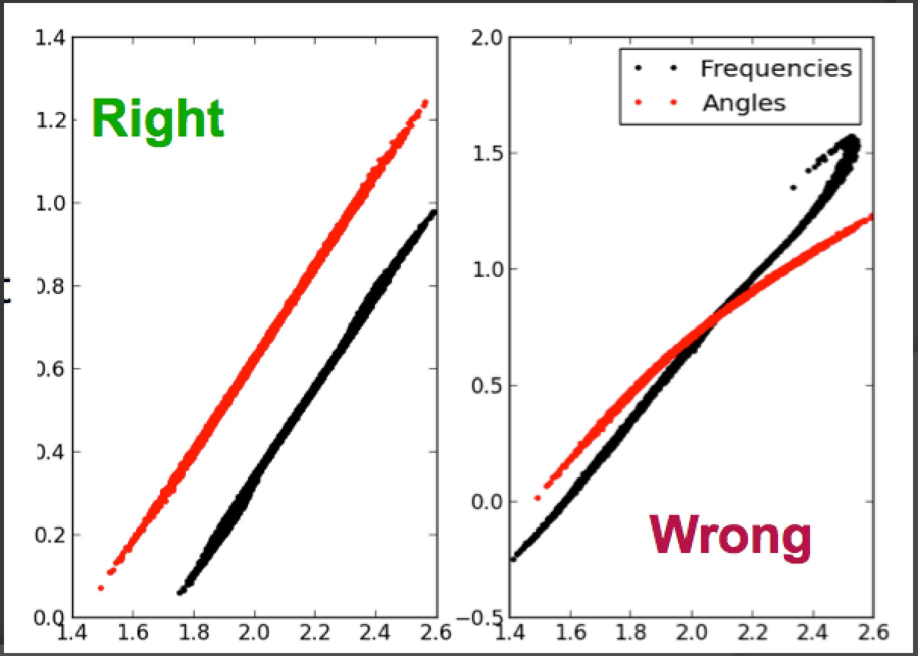


Streams aren't on simple orbit paths

Even cold streams aren't – spread in $\underline{J}$ may be very small, but for stars in stream $\underline{\theta}-\underline{\theta}_0 \neq \underline{\Omega}_0 t$ ($\underline{\Omega}_0$ frequency of progenitor)

Instead $\underline{\theta}-\underline{\theta}_0 \approx (\underline{\Omega}-\underline{\Omega}_0)t$

Can use this to determine Galactic potential from a stream.



Eyre & Binney (2011), Sanders & Binney (2013)

Future work

- We are applying the potential finding methods to RAVE data – requires further work to use sensible f($\underline{J}$,[Fe/H]).
- Torus modelling software to be released soon.
- Have shown value of $\underline{J}(\underline{x},\underline{v})$ methods for analysis, but Torus modelling (which has other advantages) is $\underline{x},\underline{v}(\underline{J},\theta)$.
- Possibility of interpolation between tori as $\underline{J}(\underline{x},\underline{v})$
- This also opens up the possibility of perturbation theory.

Conclusions

Actions & angles ($\underline{J}$,$\underline{\theta}$) are excellent ways of describing orbits

There are many ways of find actions & angles approximately in Galactic potentials

Torus modelling is a systematic procedure for accessing $\underline{J}$,$\underline{\theta}$ but not directly from $\underline{x}$,$\underline{v}$

Interpolation between tori may be an answer